A comment on "Radiation reaction reexamined: bound momentum and Schott term" by D.V. Gal'tsov and P. Spirin

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In recent e-print [1], Gal'tsov and Spirin discuss the radiation reaction in $d = 4$ classical electrodynamics of point charges. In particular, reexamining the well known regularization procedure for the retarded Green's function, the authors cast doubt on the validity of our previous results from the paper [2] where the Lorentz-Dirac equation was derived in $d > 4$, as well as from the paper [3] devoted to the radiation reaction in $d = 4$ dynamics of the massless point particle. They claim

"... the regularization proposed in [2] fails to reproduce the correct result already in the case of four dimensions, so the validity of the equations derived in [2], [3] (and of the regularization itself) is questionable."

The authors justify this statement in a peculiar way. They *incorrectly reproduce formula* (23) *from our paper* [2] (or the same formula (19) in [3]) for the derivative of the delta function on the semi-axis. The correct expression from [2], [3] reads

$$\delta'(s) = -\lim_{a\to+0}\frac{\partial}{\partial a}\frac{e^{-s/a}}{a}\,, \quad s \geq 0\,. \qquad (1)$$

In [1] it is taken without the overall minus sign. Using the incorrect formula, Gal'tsov and Spirin then industriously proceed to derive (providing a lot of details, see relations (38-42) in [1] and related explanations) the radiation reaction force in $d = 4$ with the wrong sign, directly inherited from the minus they miss in (1). This is what appears to them a "disproof" of the regularization we use and the equations we get in $d > 4$.

It was a simple exercise to check the sign in the representation for $\delta'(s)$ (1) (even though it is a common knowledge, our paper reminds how such expressions are derived), but Gal'tsov and Spirin did not choose to do that. It is probable they automatically copied this formula from our earlier e-print hep-th/0201046 where the overall sign factor had accidentally dropped out from this elementary relation. Although this minor inaccuracy had been corrected in the journal version published two years ago [2] (and which is referred to by Gal'tsov and Spirin), we considered this sign factor so obvious that we did not regard this misprint as a sufficient reason for replacing the paper in the arXiv. In the next paper [3], published a year ago and also cited (again in relation with the regularization of the delta function derivative) by Gal'tsov and Spirin, the sign was correct both in the journal and in the e-print version.

It would be appropriate to mention here that the expression for the Lorentz-Dirac force in any even dimension first obtained in our work [2], checks well with the particular result for d=6 derived in [4] by a completely different method. And, of course, our result reproduces the text-book answer for d=4.

Although the Gal'tsov-Spirin e-print contains no new results (it just compares well known achievements of the former times with each other), the authors promise to shed new light on the radiation reaction problem in higher dimensions elsewhere by applying regularizations they prefer to any other scheme: the point splitting one and/or that based on the integration of the energy-momentum flux through a hypersurface encircling the particle's world-line. It might be further appropriate to note that the result of the renormalization of singular integrals involving generalized functions of one variable, like $\delta(s^2)$ and its derivatives, can not depend on a particular choice of the regularization [5], unlike the many variable case usual in the quantum field theory. This leaves no room for ambiguity in the values like the radiation reaction force in classical electrodynamics (see also the comments in Section 3 of our paper [2]). The regularization we used in [2] has the technical convenience because it is explicitly reparametrization invariant and it allows to get the radiation reaction force in any even dimension by a mere expansion of a simple generating function in the regularization parameter. Another regularization can have other technical advantages, but it will inevitably result in the same equations as we get in [2].

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[1] D.V. Gal'tsov and P. Spirin, *"Radiation reaction reexamined: bound momentum and Schott term"*, hep-th/0405121.

[2] P.O. Kazinski, S.L. Lyakhovich and A.A. Sharapov, *"Radiation reaction and renormalization in classical electrodynamics of point particle in any dimension"*, Phys.Rev., D66 (2002) 025017.

[3] P.O. Kazinski and A.A. Sharapov, *"Radiation reaction for a massless charged particle"*, Class. Quantum Grav. 20 (2003) 2715-2725.

[4] B. P. Kosyakov, Teor. Mat. Fiz. V. 119, N. 1 (1999) 119-136 (in russian); hep-th/0207217.

[5] I.M. Gelfand and G.E. Shilov, *Generalized functions*, Vol.1 (Academic, New York, 1964).